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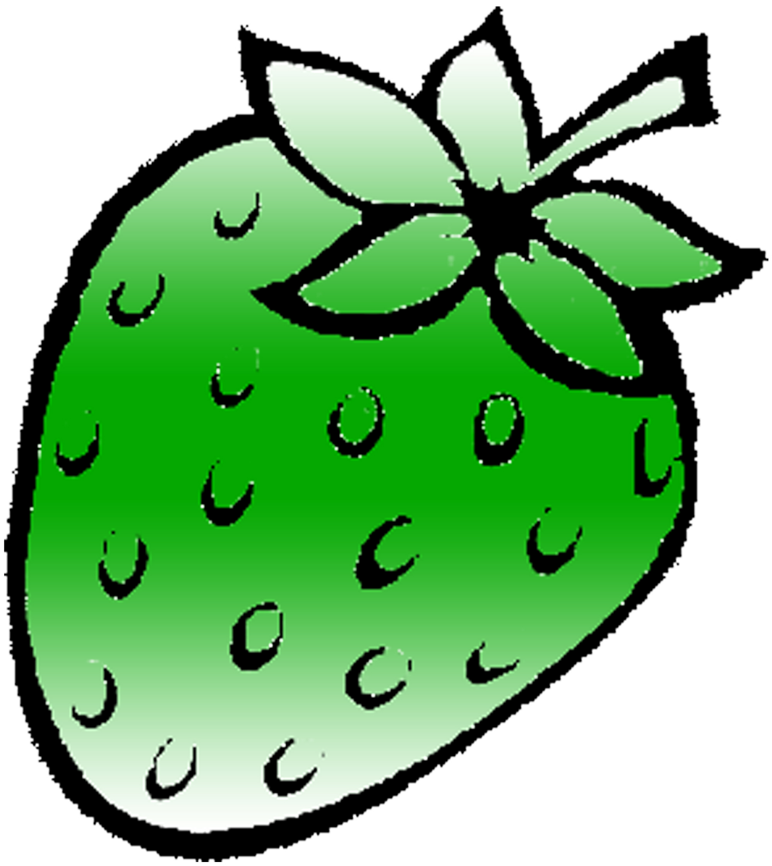

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SVKM's NMIMS

Mukesh Patel School of Technology Management & Engineering, Vile Parle, Mumbai- 400056

Tutorial Manual

Academic Year : 2014-15

Program: B. Tech / MBA (Tech) (All programs)

Semester – II

Course: - Engineering Mathematics II

SVKM'S NMIMS

Mukesh Patel School of Technology Management & Engineering

First Year

Course: Engineering Mathematics II

Course Objectives:

- To provide an understanding of Engineering Mathematics with basic concepts and their application in technical subjects.
- Application of the concepts to solve engineering problems

Course Outcomes:

After the successful completion of this course, the student will be able to :

1. Solve problems using matrix method.
2. Employ Beta, Gamma techniques to solve integrals by selecting appropriate substitution.
3. Analyse suitable method to solve differential equations.
4. Use concepts of multiple integrals to solve problems, compute area, mass of a lamina and volume.

LIST OF TUTORIALS

CO mapping with Tutorials

Tutorial No.	CO1	CO2	CO3	CO4
1	√			
2		√		
3		√		
4			√	
5			√	
6			√	
7			√	
8				√
9				√
10				√
11				√
12				√

Tutorial 1

Types of Matrices, Adjoint and Rank

1. Matrices A, B are such that $3A-2B=\begin{pmatrix}2 & 1\\ -2 & -1\end{pmatrix}$ and $-4A+B=\begin{pmatrix}-1 & 2\\ -4 & 3\end{pmatrix}$ Find A, B
2. Given $3\begin{pmatrix}x & y\\ z & w\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2w\end{pmatrix}+\begin{pmatrix}4 & y+x\\ z+w & 3\end{pmatrix}$ Find x, y, z and w
3. Express the following matrices as sum of symmetric and skew symmetric matrices.
$\begin{pmatrix}2 & -4 & 9\\ 14 & 7 & 13\\ 3 & 5 & 11\end{pmatrix}, \begin{pmatrix}2 & 0 & 5 & 3\\ 4 & 1 & 6 & 1\\ -3 & 2 & 7 & 1\\ 1 & -4 & 2 & 0\end{pmatrix}, \begin{pmatrix}1 & 0 & 5 & 3\\ -2 & 1 & 6 & 1\\ 3 & 2 & 7 & 1\\ 4 & -4 & 2 & 0\end{pmatrix}$
4. Express the following matrices as sum of Hermitian and skew hermitian matrices.
$\begin{pmatrix}2 & -3i & 1-i\\ 2-3i & 4i & -5\\ 4+i & 1+i & -3i\end{pmatrix}, \begin{pmatrix}2+3i & 0 & 4i\\ 5 & i & 8\\ 1-i & -3+i & 6\end{pmatrix}, \begin{pmatrix}2 & -3i & 1-i\\ 2-3i & 4i & -5\\ 4+i & 1+i & -3i\end{pmatrix}$
5. Express the following matrices as $P+iQ$ where P and Q both are Hermitian matrices.
$\begin{pmatrix}1+2i & 2 & 3-i\\ 2+3i & 2i & 1-2i\\ 1+i & 0 & 3+2i\end{pmatrix}, \begin{pmatrix}2i & -3 & 1-i\\ 0 & 2+3i & 1+i\\ -3i & 3+2i & 2-5i\end{pmatrix}, \begin{pmatrix}2 & 3-i & 1-i\\ 2-i & 3+i & 2+i\\ 1+i & 0 & -3i\end{pmatrix}$
6. Express the following Hermitian matrices as $P+iQ$ where P is real symmetric and Q is real skew symmetric. $\begin{pmatrix}2 & 2+i & -2i\\ 2-i & 3 & i\\ 2i & -i & 1\end{pmatrix}, \begin{pmatrix}1 & -i & 1+i\\ i & 0 & 2-3i\\ 1-i & 2+3i & 2\end{pmatrix}, \begin{pmatrix}1 & 2+i & -1+i\\ 2-i & 1 & 2i\\ -1-i & -2i & 0\end{pmatrix}$
7. Express the following Skew Hermitian matrices as $P+iQ$ where P is real skew symmetric and Q is real symmetric. $\begin{pmatrix}i & 1-i & 2+3i\\ -1-i & 2i & -3i\\ -2+3i & -3i & -i\end{pmatrix}, \begin{pmatrix}3i & -1+i & 3-2i\\ 1+i & -i & 1+2i\\ -3-2i & -1+2i & 0\end{pmatrix}$
$, \begin{pmatrix}i & 2i & -1+3i\\ 2i & 2i & 2-i\\ 1+3i & -2-i & 3i\end{pmatrix}$
8. Verify that adj(adj) A=A for A=$\begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix}$. Does it happen for every 2X2 matrix? Explain.

9. Find the adjoint and hence the inverse (if it exists):

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

10. *If* $A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$, *find* A^{-2}. Use adjoint method.

11. Find the condition on k such that the matrix $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & k & 6 \\ -1 & 5 & 1 \end{pmatrix}$ has an inverse. Obtain A^{-1} for $k = 1$

12. . Find the matrix A , if $adj.A = \begin{pmatrix} -2 & 1 & 3 \\ -2 & -3 & 11 \\ 2 & 1 & -5 \end{pmatrix}$

13. Verify that $adj.(adj.A) = A|A|$, *also find* $(adj.A)^{-1}$; *where* $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$

14. Prove that the following matrices are orthogonal and hence find A^{-1}.

$$A = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad ; \quad A = \frac{1}{\sqrt{6}} \begin{bmatrix} 0 & \sqrt{3} & \sqrt{3} \\ 2 & 1 & -1 \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

15. If $A = \frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$ is orthogonal, find a, b, c.

16. Reduce the following matrices to Row Echelon form and find its rank.

a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 3 & 5 & 9 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{pmatrix}$

17. Reduce the following matrices to normal form and find their ranks:

(i) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$ (v) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

Tutorial 2

Gamma Function and its application to solving Integrals

Solve the following:

1. Find $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx$
2. Find $\int_{-\infty}^{\infty} e^{-\frac{x^2}{4}} dx$
3. Evaluate $\int_{0}^{\infty} x^{n-1} e^{-h^2 x^3} dx$
4. Evaluate $\int_{0}^{\infty} \sqrt{x} e^{-x^3} dx$
5. Prove that $\int_{0}^{\infty} x e^{-x^8} dx \int_{0}^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$
6. Prove that $\int_{0}^{\infty} \sqrt{x} e^{-x^2} dx \int_{0}^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$
7. Prove that $\int_{0}^{\infty} \frac{e^{-x^3}}{\sqrt{x}} dx \int_{0}^{\infty} y^4 e^{-y^6} dy = \frac{\pi}{9}$.
8. Evaluate $\int_{0}^{\infty} \left(x^2 - 4\right) e^{-2x^2} dx$
9. Evaluate $\int_{0}^{1} \left(x \log x\right)^4 dx$
10. Evaluate $\int_{0}^{1} \frac{1}{(x \log 1/x)^{1/2}} dx$
11. Evaluate $\int_{0}^{1} \sqrt{\log(1/x)} dx$
12. Evaluate $\int_{0}^{1} \sqrt{x \log\left(\frac{1}{x}\right)} dx$
13. Evaluate $\int_{0}^{1} \sqrt[3]{\log\left(\frac{1}{x}\right)} dx$
14. Evaluate $\int_{0}^{\infty} \frac{x^5}{5^x} dx$

15. Evaluate $\int_0^\infty \frac{1}{3^{4z^2}}\,dz$

16. Prove that $1.3.5...(2n-1)=\frac{2^n\,\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$

17. Evaluate $\int_0^\infty \cos\left(ax^{\frac{1}{n}}\right)dx$

18. Prove that $\int_0^\infty xe^{-ax}\sin bx\,dx=\frac{2ab}{\left(a^2+b^2\right)^2}$

Tutorial 3

Beta Function and its application to solving Integrals

1. Evaluate $\int_0^9 x^{\frac{3}{2}}(9-x)^{\frac{1}{2}}\,dx$
2. Prove tat $\int_0^1 x\sqrt[3]{8-x^3}\,dx = \frac{8}{3}\beta\left(\frac{2}{3},\frac{4}{3}\right)$
3. Evaluate $\int_0^3 x^3\sqrt[5]{3-x}\,dx$
4. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^8}}\,dx$
5. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}}\,dx \quad \int_0^1 x^6(1-x)^{1/2}\,dx$
6. Evaluate $\int_0^1 x^2\left(1-x^2\right)^4\,dx$
7. Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \int_0^1 \frac{1}{\sqrt{1-x^4}}\,dx = \frac{\pi}{4}$
8. Evaluate $\int_0^1 \sqrt{\sqrt{x}-x}\,dx$
9. Prove that $\int_0^a x^6\left(a^4-x^4\right)^{1/4}\,dx = \frac{3\sqrt{2}}{128}a^8\pi$
10. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^m}}\,dx$
11. Evaluate $\int_0^1 x^4\cos^{-1}x\,dx$
12. Evaluate $\int_0^1 x^6\sin^{-1}x\,dx$
13. Evaluate $\int_0^1 \frac{x^7}{\sqrt{1-x^4}}\,dx$
14. Evaluate $\int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\,dx$

15. Evaluate $\int_0^{\pi} (1+\cos\theta)^3 \, d\theta$

16. Evaluate $\int_0^{\pi/4} (1+\cos 4\theta)^5 \, d\theta$

17. Evaluate $\int_0^{\pi/4} \cos^7 2\theta d\theta$

18. Evaluate $\int_0^{\pi/6} \cos^3 3\theta \sin^2 6\theta d\theta$

19. Evaluate $\int_0^{\pi} \sin^2\theta (1+\cos\theta)^4 \, d\theta$

20. Evaluate $\int_{-\pi/2}^{\pi/2} \cos^3\theta \, (1+\sin\theta)^2 d\theta$

21. Evaluate $\int_{-\pi}^{\pi} \sin^4 x \cos^2 x dx$

22. Prove that $\int_0^{\pi/2} \sin^p x dx \ \int_0^{\pi/2} \sin^{p+1} x dx = \dfrac{\pi}{2(p+1)}$.

23. Prove that $\int_0^{\pi} x \sin^7 x \cos^4 x dx = \dfrac{16\pi}{1155}$

24. Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)} \, dx$

25. Evaluate $\int_5^9 \sqrt[4]{(9-x)(x-5)} \, dx$

26. Evaluate $\int_0^{\infty} \dfrac{\sqrt{x}}{4+4x+x^2} dx$

27. Evaluate $\int_0^{\infty} \dfrac{\sqrt{x}}{9+6x+x^2} dx$

28. Prove that $\int_0^{\infty} \dfrac{x^8 (1-x^6)}{(1+x)^{24}} dx = 0$

29. Prove that $\int_0^{\infty} \dfrac{x^4 (1+x^5)}{(1+x)^{15}} dx = \dfrac{1}{5005}$

30. Prove that $\int_0^{\infty} \dfrac{dx}{1+x^4} = \dfrac{\pi\sqrt{2}}{4}$.

31. Evaluate $\int_0^1 \frac{x^3 - 2x^4 + x^5}{(1+x)^7} dx$

32. Prove that $\int_0^1 \frac{(1-x^4)^{3/4}}{(1+x^4)^2} dx = \frac{1}{4}\frac{1}{2^{1/4}}\beta\left(\frac{7}{4},\frac{1}{4}\right)$

Tutorial 4

Exact Differential Equations, Non Exact Differential Equation reducible to Exact form

Solve the following Differential equations :

1. $(\sin x \cos y + e^{2x})\,dx + (\cos x \sin y + \tan y)\,dy = o$
2. $(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = o$
3. $\left\{(\cos x)\log_e(2y-8) + \frac{1}{x}\right\}dx + \frac{\sin x}{y-4}dy = 0 \; ; y(1) = \frac{9}{2}$
4. $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = o$
5. $2(1 + x^2\sqrt{y})\,ydx + (x^2\sqrt{y} + 2)\,xdy = o$
6. $\frac{y}{x^2}\cos\left(\frac{y}{x}\right)dx - \frac{1}{x}\cos\left(\frac{y}{x}\right)dy + 2xdx = o$
7. $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = o$ given that $y(0) = 4$
8. $\left(y\left(1 + \frac{1}{x}\right) + \cos y\right)dx + (x + \log x - x\sin y)\,dy = o$
9. $\left(\log\left(x^2 + y^2\right) + \frac{2x^2}{x^2 + y^2}\right)dx + \left(\frac{2xy}{x^2 + y^2}\right)dy = 0$
10. $(x^2 - x\tan^2 y + \sec^2 y)\,dy = (\tan y - 2xy - y)dx$

Solve the following Differential equations :

1. $\left(x^2 + y^2\right)dx - 2xy\,dy = o$
2. $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}\left(x + xy^2\right)dy = o$
3. $\left(2x\log x - xy^2\right)dx + 2y\,dy = o$

4. $\left(y-2x^{2}\right)dx-x\left(1-xy\right)dy=o$
5. $(y-2x^{3})dx-x(1-xy)dy=0$
6. $(2x\log x-xy)dy+2ydx=0$
7. $(4xy+3y^{2}-x)dx+x(x+2y)dy=0$
8. $(xy^{2}-e^{\frac{1}{x^{3}}})dx-x^{2}ydy=0$
9. $(x^{3}e^{x}-my^{2})dx+mxydy=0$
10. $(x^{4}+y^{4})dx-xy^{3}dy=0$
11. $\left(6x^{2}+4y^{3}+12y\right)dx+3x\left(1+y^{2}\right)dy=0$

Solve the following Differential equations :

1. $\left(y^{4}+2y\right)dx+\left(xy^{3}+2y^{4}-4x\right)dy=o$
2. $\left(2xy^{2}-y\right)dx+xdy=o$
3. $\left(2xy^{4}e^{y}+2xy^{3}+y\right)dx+\left(x^{2}y^{4}e^{y}-x^{2}y^{2}-3x\right)dy=o$
4. $y\left(xy+e^{x}\right)dx-e^{x}dy=o$
5. $(xy^{3}+y)dx+2(x^{2}y^{2}+x+y^{4})dy=0$
6. $(3x^{2}y^{4}+2xy)dx+(2x^{3}y^{3}-x^{2})dy=0$
7. $y(x^{2}y+e^{x})dx-e^{x}dy=0$
8. $(2x^{2}y+e^{x})ydx-(e^{x}+y^{3})dy=0$
9. $xe^{x}(dx-dy)+e^{x}dx+ye^{y}dy=0$
10. $\left(x+2y^{3}\right)\frac{dy}{dx}=y$

Solve the following Differential equations :

1. $y\left(1+xy+xy^{2}\right)dx-x\left(1-xy+x^{2}y^{2}\right)dy=o$

2. $y\left(\sin xy+xy\cos xy\right)dx+x\left(xy\cos xy-\sin xy\right)dy=o$
3. $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$
4. $y(1+xy)dx+x(1+xy+x^2y^2)dy=0$
5. $\frac{dy}{dx}=-\frac{x^2y^3+2y}{2x-2x^3y^2}$
6. $y(1+xy+x^2y^2)dx+x(1-xy+x^2y^2)dy=0$
7. $y(1+xy)dx-x(1-xy)dy=0$
8. $y(xy+2x^2y^2)dx+x(xy+x^2y^2)dy=0$
9. $y(1+xy+x^2y^2+x^3y^3)dx+x(1-xy-x^2y^2+x^3y^3)dy=0$
10. $y(x+y)dx-x\left(y-x\right)dy=o$

Solve the following Differential equations :

1. $\left(x^2y-2xy^2\right)dx-\left(x^3-3x^2y\right)dy=o$
2. $y(x+y)dx-x\left(y-x\right)dy=o$
3. $\left(x^4+y^4\right)dx-xy^3\,dy=o$
4. $\left(x^2+y^2+1\right)dx-2xy\,dy=o$
5. $(x^2+y^2)dx-(x^2+xy)dy=0$
6. $(x^2-xy+y^2)dx-xy\,dy=0$
7. $x^2ydx-(x^3+y^3)dy=0$
8. $(3xy^2-y^3)dx-(2x^2y-xy^2)dy=0$
9. $(x^3+y^3)dx-xy^2dy=0$
10. $x(x-y)dy+y^2dx=0$

Tutorial 5
Linear Differential Equations, Equations Reducible To Linear Form, Bernoulli's Equations

Solve the following Differential equations :

1. $xdy - (y - x)\,dx = o$

2. $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$

3. $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$

4. $\sin 2x \dfrac{dy}{dx} = y + \tan x$

5. $\dfrac{dy}{dx} + \dfrac{4x}{x^2+1}.y = \dfrac{1}{(x^2+1)^3}$

6. $x\cos x \dfrac{dy}{dx} + y(x\sin x + \cos x) = 1$

7. $\dfrac{dy}{dx} + \left(\dfrac{1-2x}{x^2}\right) y = 1$

8. $(1 - x^2)\dfrac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$

9. $x(x-1)\dfrac{dy}{dx} - (x-2)\,y = x^3(2x-1)$

10. $(1 + x + xy^2)dy + (y + y^3)dx = 0$

11. $(1 + y^2)dx = \left(e^{\tan^{-1} y} - x\right)dy$

12. $(y+1)dx + \left[x - (y+2)e^y\right]dy = 0$

13. $(1 + \sin y)\dfrac{dx}{dy} = [2y\cos y - x(\sec y + \tan y)]$

Solve the following Differential equations :

1. $\dfrac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

2. $\tan y\dfrac{dy}{dx}+\tan x=\cos y.\cos^3 x$

3. $e^x(x+1)dx+(ye^y-xe^x)dy=0$

4. $\dfrac{dy}{dx}=e^{x-y}.(e^x-e^y)$

5. $\dfrac{dy}{dx}+x^3\sin^2 y+x\sin 2y=x^3$

6. $\dfrac{dy}{dx}+(2x\tan^{-1} y-x^3)(1+y^2)=0$

7. $y\dfrac{dy}{dx}+\dfrac{4x}{3}-\dfrac{y^2}{3x}=0$

8. $\dfrac{dy}{dx}=1-2x(y-x)+x^3$

9. $\dfrac{dy}{dx}-\dfrac{\tan y}{1+x}=(1+x)e^x\sec y$

10. $\sec y\dfrac{dy}{dx}+2x\sin y=2x\cos y$

11. $\tan y\dfrac{dy}{dx}+\tan x\left(1-\cos y\right)=0$

12. $\dfrac{dy}{dx}=\dfrac{e^y}{x^2}-\dfrac{1}{x}$

Solve the following Differential equations :

1. $x\dfrac{dy}{dx}+y=y^6x^3$

2. $xy\left(1+xy^2\right)\dfrac{dy}{dx}=1$

3. $4x^2y\,\dfrac{dy}{dx}=3x\left(3y^2+2\right)+\left(3y^2+2\right)^3$

4. $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$

5. $\dfrac{dy}{dx}=x^3y^3-xy$

6. $y-\cos x\dfrac{dy}{dx}=y^2(1-\sin x)\cos x$

7. $\dfrac{dy}{dx} = xy + y^2 e^{(-x^2/2)}.\log x$

8. $\dfrac{dy}{dx} + \left(\dfrac{x}{1-x^2}\right) y = x\sqrt{y}$

9. $\dfrac{dy}{dx} + \dfrac{2}{x} y = \dfrac{y^3}{x^3}$

10. $ydx + x(1-3x^2y^2)dy = 0$

11. $y\dfrac{dx}{dy} = x - yx^2 \sin y$

12. $4xy\dfrac{dy}{dx} = (y^2+3) + x^3(y^2+3)^3$

Tutorial 6

Linear homogeneous differential equation with constant coefficients, Non-homogeneous linear differential equation with constant Coefficients

Solve the following differential equations :

1. $2\frac{d^2y}{dx^2}+5\frac{dy}{dx}-12y=0$
2. $6\frac{d^3x}{dt^3}+23\frac{d^2x}{dt^2}+\frac{dx}{dt}+12x=0$
3. $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=0$
4. $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=0$
5. $\frac{d^3y}{dx^3}+8y=0$
6. $\frac{d^4y}{dx^4}-2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=0$
7. $\left\{(D-1)^4\,(D^2+2D+2)^2\right\}y=0$

Solve the following differential equations :

1. $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-x}$
2. $6\frac{d^2y}{dx^2}+17\frac{dy}{dx}+12y=e^{\frac{-3x}{2}}+2^x$
3. $\left(D^3-2D^2-5D+6\right)y=e^{3x}+8$
4. $\left(D^2-D+1\right)y=\cos 2x$
5. $\left(D^4+D^2+16\right)y=\sin^2 x$
6. $D^2\left(D^2+1\right)y=\sin x+e^{-x}$
7. $\left(D^4+10D^2+9\right)y=\cos\left(2x+3\right)$
8. $\left(D^3+D\right)y=\cos x$

9. $\operatorname{cosec} x \dfrac{d^4 y}{dx^4} + \operatorname{cosec} x . y = \sin 2x$

10. $\left(D^3 + 2D^2 + D\right) y = x^2 + x$

11. $\left(D^2 + 4\right) y = x^2 + \sin 2x$

12. $\left(D^2 - 4D + 4\right) y = 8\left(x^2 + \sin 2x + e^{2x}\right)$

13. $\left(D^2 + 5D + 6\right) y = x^2 + e^{-2x}$

14. $\dfrac{d^3 y}{dx^3} - 2\dfrac{dy}{dx} + 4y = 3x^2 - 5x + 2$

15. $\left(D^3 - D^2 - 6D\right) y = x^2 + 1$

16. $\left(D^2 - 4D + 4\right) y = e^{2x} . x^2$

17. $\left(D^3 - 7D - 6\right) y = \left(1 + x^2\right) e^{2x}$

18. $\left(D^3 - 7D - 6\right) y = e^{2x} (x + 1)$

19. $\left(D^3 - 3D^2 + 3D - 1\right) y = xe^x + e^x$

20. $\dfrac{d^2 y}{dx^2} + 2y = x^2 e^{3x} + e^x - \cos 2x$

21. $\left(D^2 - 1\right) y = \cosh x \cos x$

22. $\left(D^2 - 1\right) y = x \sin 3x$

23. $\left(D^2 - 1\right) y = x^2 \sin 3x$

24. $\left(D^4 + 2D^2 + 1\right) y = x^2 \sin x$

25. $\left(D^4 + 4\right) y = x \sin^2 x$

26. $\left(D^4 - 4\right) y = x \sinh x$

27. $\left(D^2 + 3D + 2\right) y = 3x^2 + 2e^{3x}$

28. $\left(D^2 - 2D\right) y = e^x \sin 3x$

29. $\left(D^2 - 1\right) y = e^x \sin 2x$

30. $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 10e^{3x} + 4x^2$

Tutorial 7

Method of Variation of Parameters, Equation reducible to Linear Differential Equation with constant coefficients

Solve following differential equations using variation of parameters method :

1. $\frac{d^2y}{dx^2}+k^2y=\tan kx$
2. $\left(D^2-1\right)y=\frac{2}{1+e^x}$
3. $\left(D^2-1\right)y=e^x\sin x$
4. $\left(D^2-6D+9\right)y=\frac{e^{3x}}{x^2}$
5. $\frac{d^2y}{dx^2}+a^2y=\sec ax$

Solve following differential equations :

1. $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=\sin(\log x)$
2. $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^{-1}$
3. $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=8+4\log x$ (Hint: multiply throughout by x^2 to get Cauchy's D.E.)
4. $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$
5. $x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2\log x$ (Hint: multiply throughout by x to get Cauchy's D.E.)
6. $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1+x)^2}$

Solve following differential equations :

1. $(5+2x)\dfrac{d^2y}{dx^2}-6(5+2x)\dfrac{dy}{dx}+8y=0$

2. $(x+a)^2\dfrac{d^2y}{dx^2}-4(x+a)\dfrac{dy}{dx}+6y=x$

3. $(3x+1)^2\dfrac{d^2y}{dx^2}-3(3x+1)\dfrac{dy}{dx}-12y=9x$

4. $(2x+1)^2\dfrac{d^2y}{dx^2}-2(2x+1)\dfrac{dy}{dx}-12y=6x$

5. $(x+2)^2\dfrac{d^2y}{dx^2}-(x+2)\dfrac{dy}{dx}+y=3x+4$

Tutorial 8

Evaluation of Double Integrals in Cartesian and Polar Coordinate systems

1. Evaluate the following Double integrals (Cartesian Coordinates):

i) $\int_0^1 \int_0^{\frac{1}{y}} y e^{xy} dx dy$

ii) $\int_0^1 \int_{y^2}^{y} (1+xy^2) dx dy$

iii) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} xy dy dx$

iv) $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{x} \cos(x+y) dy dx$

v) $\int_0^5 \int_{2-x}^{2+x} dy dx$

vi) $\int_0^1 \int_0^x (x^2+y^2) x dy dx$

vii) $\int_0^{\frac{\pi}{2}} \int_0^y \cos 2y \sqrt{1-a^2 \sin^2 x} dx dy$

viii) $\int_0^1 \int_x^{1/x} \frac{y}{(1+xy)^2(1+y^2)} dy dx$

ix) $\int_0^a \int_0^x \frac{e^y}{\sqrt{(a-x)(x-y)}} dy dx$.

x) $\int_0^1 \int_{x^2}^{2-x} y dy dx$

xi) $\int_0^1 \int_{-\sqrt{y}}^{-y^2} xy dy dx$

xii) $\int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x dx dy}{y^2+x^2+a^2}$

xiii) $\int_0^1 \int_{x^2}^{x} x y (x+y) dy dx$

xiv) $\int_0^a \int_0^x \frac{e^y}{\sqrt{(a-x)(x-y)}} d y d x$

xv) $\int_0^2 \int_{\sqrt{2x}}^{2} \frac{y^2 dy dx}{\sqrt{y^4-4x^2}}$

xvi) $\int_0^a \int_{2\sqrt{ax}}^{\sqrt{5xa-x^2}} \frac{\sqrt{x^2+y^2}}{y^2} dx dy$

xvii) $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2) dx dy$

xviii) $\int_0^2 \int_{1-\sqrt{2x-x^2}}^{1+\sqrt{2x+x^2}} \frac{dx dy}{(x^2+y^2)^2}$

xix) $\int_0^a \int_y^{a+\sqrt{a^2-y^2}} \frac{dx dy}{(4a^2+x^2+y^2)^2}$

xx) $\int_0^{4a} \int_{y^2/4a}^{y} \frac{(x^2-y^2)}{(x^2+y^2)} dx dy$

2. Evaluate the following Double integrals (Polar Coordinates):

i) $\int_{0}^{\frac{\pi}{2}} \int_{0}^{a\cos\theta} r\sqrt{a^2 - r^2}\,dr\,d\theta$

ii) $\int_{0}^{\frac{\pi}{2}} \int_{0}^{a(1+\sin\theta)} r^2 \cos\theta\,dr\,d\theta$

iii) $\int_{0}^{\frac{\pi}{4}} \int_{0}^{\sqrt{\cos 2\theta}} \frac{r}{\left(1+r^2\right)^2}\,dr\,d\theta$

iv) $\int_{0}^{\pi/2} \int_{0}^{2a\cos\theta} r^2 \sin\theta\,dr\,d\theta$

v) $\int_{0}^{\pi} \int_{2\sin\theta}^{4\sin\theta} r^3\,dr\,d\theta$

Tutorial 9

Evaluation of Double Integration over a region

1. Evaluate $\iint_R xy\,dxdy$ over the region R given by $x^2 + y^2 = a^2, x \geq 0$ *and* $y \geq 0$.
2. Evaluate $\iint \left(x^2 + y^2\right) dxdy$ throughout the area enclosed by the curves $y = 4x, x + y = 3, y = 0$ *and* $y = 2$.
3. Evaluate $\iint \left(x^2 + y^2\right) dxdy$ over the region area of the triangle whose vertices are $(0,1),(1,0),(1,2)$.
4. Evaluate $\iint_R xy\,dxdy$ over the region R given by $y^2 = 4x$ *and* $y = 2x - 4$.
5. $\iint_R \frac{y\sqrt{x}}{\sqrt{(1-x^2)(1-y^2)}}\,dx\,dy$ where R is the region in the first quadrant bounded by, $y^2 = x,\ x+1$ *and* $y = 0$.
6. $\iint_R xy\,(x-1)\,dx\,dy$ where R is the region bounded by, $xy = 4, y = 0, x = 1$ *and* $x = 4$.
7. $\iint_R \frac{dx\,dy}{x^2 + y^2}$ over $x \geq 1, y \geq x^2$
8. $\iint_R x^2 dx\,dy$ R is region in first quadrant bounded by $y = \frac{16}{x}, x = 8, y = 0$ *and* $y = x$
9. $\iint_R x\,y\,(x+y)\,dx\,dy$ R is region bounded between $x^2 = y$ *and* $x = y$
10. $\iint_R xy\,dxdy$ where R is the region bounded by $x = y^2, x = 2 - y, y = 0$.
11. $\iint_R y\,dx\,dy$ where R is the region bounded by $y = x^2,\ x + y = 2$.
12. $\iint_R x^2 y^2 dx\,dy$. where R is the +ve quadrant of the circle $x^2 + y^2 = 1$.
13. Prove That: $\iint_R \sqrt{xy - y^2}\,dxdy = 6$ where R is triangle having vertices (0,0), (1,1) & (10,1).

14. $\iint_R ydxdy$ where R is bounded by $y = x$ & $y = 4x - x^2$.

15. $\iint_R xydxdy$ where R is bounded by $y^2 = 4ax \,\&\, x^2 = 4ay$.

16. $\iint_R xy\,(x-1)\,dx\,dy$ R is region bounded by $xy = 4, y = 0, x = 1, = 4$.

17. Evaluate $\iint_R \frac{dydx}{\sqrt{1-2x^2-y^2}}$ over the region R which is the first quadrant of the ellipse $2x^2 + y^2 = 1$.

18. $\iint_R xy\ dxdy$ where R is bounded by $y^2 + x^2 - 2x = 0, y^2 = 2x \,\&\, y = x$.

19. $\iint_R \frac{ye^{2y}}{\sqrt{(1-x)(x-y)}}dxdy$ where R is triangle formed by (0,0),(1,0),(1,1).

20. $\iint_R \sin\pi(ax+by)\ dxdy$ where R is bounded by x = 0, y = 0, ax + by =1.

21. Evaluate $\iint_A \frac{r}{\sqrt{r^2+4}}drd\theta$ where A is a loop of of $r^2 = 4\cos 2\theta$.

22. Evaluate $\iint r\sin\theta drd\theta$ over the area of the cardiode $r = a(1+\cos\theta)$.

23. Evaluate $\iint r\sqrt{a^2-r^2}drd\theta$ over the upper half of the circle $r = a\cos\theta$.

24. Evaluate $\iint_R r^3 drd\theta$ where R is area common to the circles $r = 2\sin\theta$, $r = 2\cos\theta$.

25. Evaluate $\iint_R rdrd\theta$ where R is area between $r = 1+\cos\theta$, $r = 1$.

26. Evaluate $\iint_R \sin\theta dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1+\cos\theta)$.

27. Evaluate $\iint_R \frac{rdrd\theta}{(1+r^2)^2}$ where R is area bounded $r^2 = \cos 2\theta$.

28. Evaluate $\iint_R \frac{rdrd\theta}{(a^2+r^2)^2}$ where R is area bounded within one loop of the lemniscate $r^2 = a^2\cos 2\theta$.

29. Evaluate $\iint r^2 dr d\theta$ over the area included between the circles $r = a\cos\theta, r = 2a\cos\theta$.
30. Evaluate $\iint_R r^4 \cos^3\theta dr d\theta$ where R is the interior of the circle $r = 2a\cos\theta$.

Tutorial 10

Double Integration by change of coordinate system and change of order

1. Change to polar coordinates and evaluate the following integrals:

i) $\int_0^1 \int_x^{\sqrt{2x-x^2}} \left(x^2+y^2\right) dxdy$

ii) $\int_0^{\frac{a}{\sqrt{2}}} \int_y^{\sqrt{a^2-y^{22}}} \log\left(x^2+y^2\right) dxdy$

iii) $\int_0^a \int_y^a \frac{xdydx}{x^2+y^2}$

iv) $\int_0^{2a} \int_0^{2ax-x^2} dxdy$

v) $\int_0^{1/2} \int_0^{\sqrt{x-x^2}} \frac{4xy}{x^2+y^2} e^{-x^2-y^2} dxdy$

vi) $\int_0^\infty \int_0^\infty e^{-x^2-y^2} dxdy$

vii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dxdy}{\left(1+x^2+y^2\right)^{3/2}}$

viii) $\int_0^1 \int_{\sqrt{x-x^2}}^{\sqrt{1-x^2}} \frac{xy}{x^2+y^2} e^{-x^2-y^2} dxdy$

ix) $\int_0^2 \int_{1-\sqrt{2x-x^2}}^{1+\sqrt{2x-x^2}} \frac{dxdy}{\left(x^2+y^2\right)^2}$

x) $\int_0^{4a} \int_{y^2/4a}^{y} \frac{x^2-y^2}{\left(x^2+y^2\right)} dxdy$

xi) $\int_0^3 \int_0^{\sqrt{3x}} \frac{dxdy}{\left(x^2+y^2\right)^{1/2}}$

xii) $\int_0^a \int_0^x \frac{x^3 dxdy}{\left(x^2+y^2\right)^{1/2}}$

xiii) $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sin\left[\frac{\pi}{a^2}\left(a^2-x^2-y^2\right)\right] dxdy$

xiv) $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-x^2-y^2} dxdy$

2. Transform the integral to Cartesian form and hence evaluate $\int_0^\pi \int_0^a r^3 \sin\theta cos\theta dr d\theta$.

3. Transform to polar and evaluate $\iint_R \frac{\left(x^2+y^2\right)^2}{x^2y^2} dxdy$ where R is bounded by x^2+y^2 = ax , x^2+y^2 = by , a,b >0.

4. Transform to polar and evaluate $\iint_R xy\left(x^2+y^2\right)^{3/2} dxdy$ where R is the first quadrant of the circle $x^2+y^2=a^2$.

5. Transform to polar and evaluate $\iint_R \left(x^2+y^2\right)^2 dxdy$ where R is bounded by $x^2+y^2=a^2$.
6. Transform to polar and evaluate $\iint_R \sin\left(x^2+y^2\right) dxdy$ where R is bounded by $x^2+y^2=a^2$.
7. Transform to polar and evaluate $\iint_R xy\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2 dxdy$ where R is in first quadrant bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
8. Transform to polar and evaluate $\iint_R \left(x^{m-1}y^{n-1}\right) dxdy$ where R is in first quadrant bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
9. Transform to polar and evaluate $\iint_R \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}} dxdy$ over ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
10. Evaluate $\iint_R \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dxdy$ over circle $x^2+y^2=1$.
11. Evaluate $\iint_R \frac{x^2y^2}{\left(x^2+y^2\right)} dxdy$ where R is bounded by $x^2+y^2=a^2$, $x^2+y^2=b^2$, (a>b).
12. Transform to polar and evaluate $\iint_R \frac{dxdy}{\left(1+x^2+y^2\right)^2}$ over one loop of lemniscate $\left(x^2+y^2\right)^2=x^2-y^2$.
13. Transform to polar and evaluate $\iint_R \frac{dxdy}{\sqrt{xy}}$ where R is bounded by $x^2+y^2=x$, $y\geq 0$.
14. Change the order of integration:

i) $\int_0^4 \int_{\sqrt{4x-x^2}}^{2\sqrt{x}} f(x,y)\,dxdy$

ii) $\int_{-a}^{a} \int_0^{y^2/a} f(x,y)\,dxdy$

iii) $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\,dxdy$

iv) $\int_a^b \int_a^{c^2/x} f(x,y)\,dxdy$

v) $\int_0^1 \int_{x^2}^{2-x} f(x,y)\,dxdy$

vi) $\int_0^4 \int_y^{4+\sqrt{16-y^2}} f(x,y)\,dxdy$

vii) $\int_{-2}^{3}\int_{y^2-6}^{y} f(x,y)\,dxdy$

viii) $\int_{0}^{5}\int_{2-x}^{2+x} f(x,y)\,dxdy$

ix) $\int_{0}^{k}\int_{-k-\sqrt{k^2-y^2}}^{k+\sqrt{k^2-y^2}} f(x,y)\,dxdy$

x) $\int_{0}^{4}\int_{y/2}^{9-y} f(x,y)\,dxdy$

xi) $\int_{0}^{a}\int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y)\,dxdy$

xii) $\int_{0}^{1}\int_{y^2}^{y^{1/3}} f(x,y)\,dxdy$

15. Change the order of integration and evaluate the following :

i) $\int_{0}^{a}\int_{0}^{x}\frac{dx\,dy}{(y+a)\sqrt{(a-x)(x-y)}}$

ii) $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}\frac{\cos^{-1}x}{\sqrt{1-x^2}\,\sqrt{1-x^2-y^2}}\,dx\,dy$

iii) $\int_{1}^{4}\int_{\sqrt{y}}^{2}\left(x^2+y^2\right)dx\,dy$

iv) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} y^2\,dy\,dx$

v) $\int_{0}^{a}\int_{\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$

vi) $\int_{0}^{a}\int_{0}^{\sqrt{a^2-y^2}}\frac{dx\,dy}{\left(1+e^{y}\right)\sqrt{a^2-x^2-y^2}}$

vii) $\int_{a}^{b}\int_{a-\frac{a}{b}\sqrt{b^2-y^2}}^{a^2y^2/b^2} xy\,dx\,dy$

viii) $\int_{-2}^{1}\int_{x^2}^{2-x} y\,dx\,dy$

ix) $\int_{0}^{\infty}\int_{0}^{x} xe^{-x^2/y}\,dx\,dy$

x) $\int_{0}^{1}\int_{x}^{2-x}\frac{x}{y}\,dy\,dx$

xi) $\int_{0}^{2}\int_{\sqrt{2y}}^{2}\frac{x^2}{\sqrt{x^4-4y^2}}\,dx\,dy$

xii) $\int_{0}^{1}\int_{x}^{1/x}\frac{y}{(1+xy)^2\left(1+y^2\right)}\,dy\,dx$

xiii) $\int_{0}^{a}\int_{0}^{y}\frac{dx\,dy}{\sqrt{(a^2-x^2)(a-y)(y-x)}}$

xiv) $\int_{0}^{\pi/2}\int_{0}^{y}\cos 2y\sqrt{1-a^2\sin^2 x}\,dx\,dy$

16. Express as a single integral and then evaluate $\int_{0}^{1}\int_{0}^{y}\left(x^2+y^2\right)dxdy+\int_{1}^{2}\int_{0}^{2-y}\left(x^2+y^2\right)dxdy$.

Tutorial 11

Applications of Double Integration

1. Find the area bounded by the lines $y = x + 2, y = -x + 2, x = 5$.
2. Find the area bounded by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
3. Find the area between parabola $y = x^2 - 3x$, the line $y = 2x$ & x-axis.
4. Find the area outside the parabola $y^2 = 4 - 4x$ & inside $y^2 = 4 - x$.
5. Find the area between the curves $y = ax^2$ and $y = 1 - \frac{x^2}{a}$, a>0.
6. Find the area of the curvilinear triangle lying in first quadrant, bounded by $x^2 = 4ay$, $y^2 = 4ax$ & inside $x^2 + y^2 = a^2$.
7. Find the area of the loop of the curve $2y^2 = (x-2)(x-10)^2$.
8. Find the area bounded between the curve $x\left(y^2 + a^2\right) = a^3$ and its asymptote.
9. Find the area between the rectangular hyperbola $3xy = 2$ and the line $12x + y = 6$.
10. Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.
11. Find the area of the loop of the curve $2y^2 = (x-2)(x-10)^2$.
12. Find the area bounded between the curve $x\left(y^2 + a^2\right) = a^3$ and its asymptote.
13. Find the area between the rectangular hyperbola $3xy = 2$ and the line $12x + y = 6$.
14. Find the area bounded by the curves $y = x^2$ & line $x + y = 2$.
15. Find the total area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.
16. Find the area enclosed by one loop of $x^4 + y^4 = 2a^2xy$, by converting it to polar coordinates.
17. Find the area common to $r = 2a\cos\theta$ & $r = a$.
18. Find the area common to $r = a\cos\theta$ & $r = a\sin\theta$.

19. Find the total area included between the two cardioids $r = a(1+\cos\theta)$ & $r = a(1-\cos\theta)$.
20. Find the area lying inside a cardioid $r = 1+\cos\theta$ and outside $r(1+\cos\theta) = 1$.
21. Find by double integration the area inside the circle $r = 2a\sin\theta$ & outside the cardioid $r = a(1-\cos\theta)$.
22. Find area of the curve $r = a\cos 3\theta$.
23. The larger of the two areas into which the circle $r^2 = 16^2$ is divided by the parabola $y^2 = 24x$.
24. In the cycloid $x = a(\theta+\sin\theta), y = a(1-\cos\theta)$.Find the area between the base and the portion of the curve from cusp to cusp.
25. A lamina is bounded by the curves $y = x^2 - 3x$ & $y = 2x$.If the density at any point is given by λxy. Find the mass of the lamina.
26. Find the mass of the lamina in the form of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. If the density at any point varies as the product of the distance from the axes of the ellipse.
27. The density at any point of a cardioide $r = a(1+\cos\theta)$ varies as the square of its distance from its axes of symmetry. Find its mass.
28. Find the mass of the plane in the form of one loop of lemniscate $r^2 = a^2\sin 2\theta$, if the density varies as the square of the distance from the pole.
29. The density at any point of a uniform circular lamina of radius a varies as its distance from a fixed point on the circumference of the circle. Find the mass of the lamina.

Tutorial 12

Triple Integration

Evaluation in Cartesian Coordinates

1. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$
2. Evaluate $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dxdydz$.
3. Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z} (x+y+z)\,dzdxdy$.
4. Evaluate $\iiint_R (x+y+z)\,dxdydz \quad where\ R: 0\le x\le 1, 1\le y\le 2, 2\le z\le 3$.
5. Evaluate $\iiint x^2 dxdydz$ throughout the volume of the tetrahedron $x\ge 0, y\ge 0, z\ge 0\ and\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}\le 1$.

Evaluation in Spherical Coordinates

6. Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{(a^2-r^2)/a} r\,d\theta drdz$
7. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.
8. Evaluate $\iiint_V \frac{dxdydz}{\left(x^2+y^2+z^2\right)^{3/2}}$ where V is the region bounded by the spheres $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=25$.
9. Evaluate $\iiint \left(x^2+y^2+z^2\right)^m dxdydz, m>0$ over the unit sphere.
10. Evaluate $\iiint z^2 dxdydz$ over the hemisphere $z\ge 0$, $x^2+y^2+z^2\le a^2$.
11. Evaluate $\iiint x^2 dxdydz$ over the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using spherical coordinates.

12. Show that if R is the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$$\iiint_R \sqrt{a^2b^2c^2-b^2c^2x^2-c^2a^2y^2-a^2b^2z^2}\,dxdydz=\frac{\pi^2a^2b^2c^2}{4}.$$

Evaluation in Cylindrical Coordinates

13. Evaluate $\iiint_V \left(x^2+y^2\right)dxdydz$ where V is the volume bounded by $x^2+y^2=2z$ and $z=2$.
14. Evaluate $\iiint z\left(x^2+y^2+z^2\right)dxdydz$ over the volume of the $x^2+y^2=a^2$ intercepted by the plane $z=0$ and $z=h$.

Volume

15. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.
16. Find the volume bounded by the cylinders $y^2=x, y=x^2$ and the planes $z=0, x+y+z=2$.
17. Find the volume enclosed by the sphere $x^2+y^2+z^2=16$ and the cone $x^2+y^2=z^2\tan\alpha$.
18. Find the volume of the paraboloid $x^2+y^2=4z$ cut off by the plane $z=4$.
19. Find the volume of the solid bounded by the plane $z=0$, the paraboloid $x^2+y^2+2=z$ and the cylinder $x^2+y^2=4$.
20. Find the volume of the ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

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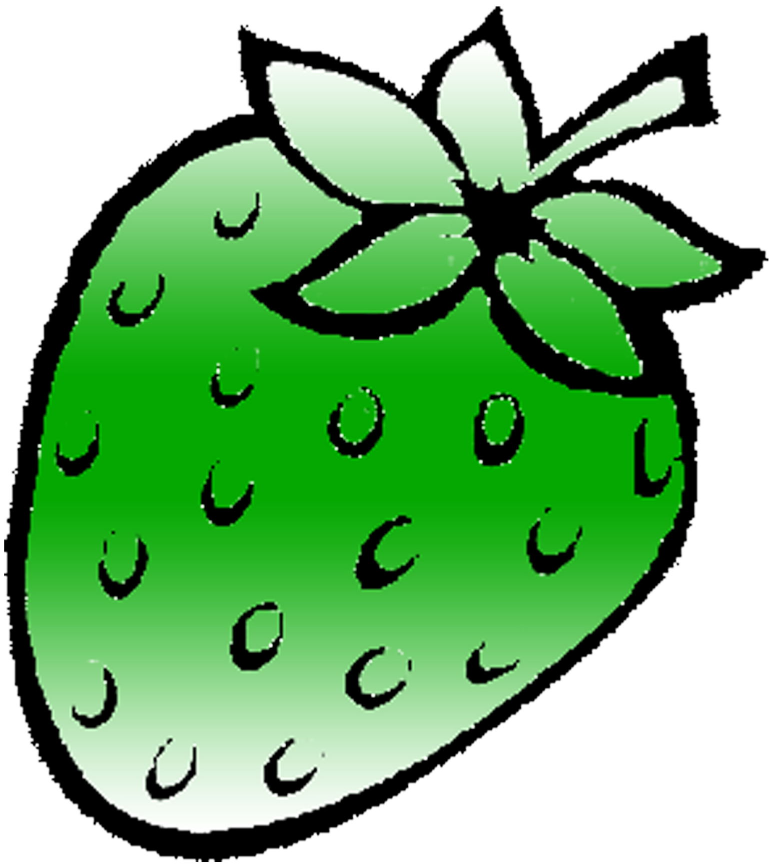

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