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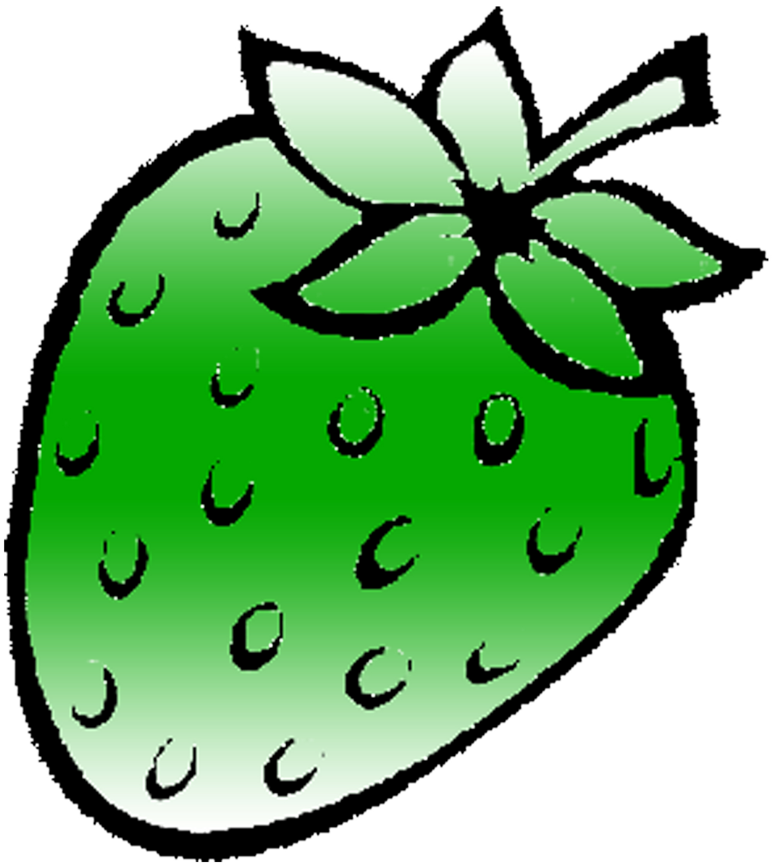

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B. Tech./ Semester-III / Engineering Mathematics-III / Lab Manual

Course outcomes:

Engineering Mathematics-III

1. Define eigen values, eigen vectors, Laplace transforms and Fourier series.
2. Extend the knowledge of matrices to reduce quadratic form to canonical form.
3. Apply Laplace transforms of commonly used functions and its properties.
4. Examine Linearly dependence and independence of the vectors.
5. Construct Fourier series for the function in the interval $[\alpha, \alpha+2\pi]$ and $[\alpha, \alpha+2c]$.
6. Solve problems using applications of Laplace transforms, matrices and Half range sine and cosine series.

TUTORIAL 1

1. Test for consistency the following equations and if possible solve them:

(i) $x+2y-z=1, \quad x+2y+2z=9, \quad 2x+y-z=2.$

(ii) $x_1-3x_2-8x_3=-10, \quad 3x_1+x_2-4x_3=0, \quad 2x_1+5x_2+6x_3=13.$

(iii) $6x+y+z=-4, \quad 2x-3y-z=0, \quad -x-7y-2z=7.$

(iv) $3x+3y+2z=1, \quad x+2y=4, \quad 10y+3z=-2, \quad 2x-3y-z=5$

(v) $x+3y-z=4, \quad 2x+y+z=7, \quad 2x-4y+4z=6, \; 3x+4y=11$

(vi) $2x_1+x_2+x_3=4, x_1-x_2+3x_3=3, 4x_1-x_2-x_3=2$

2. For what value of λ the equations $3x-2y+\lambda z=1, \quad 2x+y+z=2, \quad x+2y-\lambda z=-1$, will have no unique solution? Will the equations have any solution for this value of λ.

3. For what value of λ, the following system of equations possesses a non-trivial solution? Obtain the solution for real values of λ:

$$3x_1+x_2-\lambda x_3=0, \quad 4x_1-2x_2-3x_3=0, \quad 2\lambda x_1+4x_2+\lambda x_3=0$$

4. Investigate for what values of λ and μ the equations

(1) $x+2y+3z=4, \quad x+3y+4z=5, \quad x+3y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite no. of solutions.

(2) $2x+3y+5z=9, \quad 7x+3y-2z=8, \quad 2x+3y-\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite no. of solutions.

5. Solve the following equations:

(i) $x_1 - 2x_2 + 3x_3 = 0, \quad 2x_1 + 5x_2 + 6x_3 = 0$

(ii) $x_1 - x_2 + x_3 = 0, \quad x_1 + 2x_2 + x_3 = 0, \quad 2x_1 + x_2 + 3x_3 = 0$.

(iii) $2x_1 - x_2 + 3x_3 = 0, \quad 3x_1 + 2x_2 + x_3 = 0, \quad x_1 - 4x_2 + 5x_3 = 0$.

(iv) $x_1 + x_2 - x_3 + x_4 = 0, \quad x_1 - x_2 + 2x_3 - x_4 = 0, \quad 3x_1 + x_2 + x_4 = 0$

(v) $x_1 + 2x_2 + 3x_3 = 0, \; 2x_1 + 3x_2 + x_3 = 0, \; 4x_1 + 5x_2 + 4x_3 = 0, \; x_1 + 2x_2 - 2x_3 = 0$

TUTORIAL 2

1. Examine whether the following vectors are linearly independent or dependent. If linearly dependent find the relation between:

(i) $[1,-1,1], [2,1,1], [3,0,2]$ (ii) $[3,1,-4], [2,2,-3], [0,-4,1]$ (iii) $[1,1,1], [1,1,0], [1,0,0]$

(iv) $[2,1,1], [1,3,1], [1,2,-1]$ (v) $[1,2,1], [2,1,4], [4,5,6], [1,8,-3]$

(vi) $[1,2,-1,0], [1,3,1,2], [4,2,1,0], [6,1,0,1]$

(vii) $[1,3,4,-6], [0,1,6,0], [2,2,2,-3], [1,1,-4,-4]$, (viii) $[2,1,-1,1], [1,2,1,-1], [1,2,2,1]$

2. Find the characteristic equation, eigen values and eigen vectors of the following matrices:

(i) $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ (vi) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ (vii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(ix) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (x) $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

TUTORIAL 3

1. For the matrix $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, prove that $A^{-1}=A^2-5A+9I$.

2. For the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}$, prove that $A^{-1}=\frac{1}{40}\left[A^2+A-18I\right]$.

3. For the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$, prove that $A^{-1}=\frac{1}{4}\left[A^2-6A+9I\right]$.

4. Find the characteristic equation of the following matrices and obtain the inverse:

(i) $\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$ (iii) $\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$ (iv) $\begin{bmatrix}1 & 2 & 3\\ 3 & 1 & 1\\ 0 & 1 & 2\end{bmatrix}$

5. Find the characteristic equation of the matrix given below and verify that it satisfies Cayley-Hamilton theorem:

(i) $\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$ (ii) $\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ (iii) $\begin{bmatrix}2 & 1 & 3\\ 1 & -1 & 2\\ 1 & 2 & 1\end{bmatrix}$ (iv) $\begin{bmatrix}1 & 1 & 3\\ 1 & 0 & -3\\ -2 & 1 & 0\end{bmatrix}$

6. Find the characteristic equation of the matrix A and hence find A^{-1} and A^4.

(i) $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 2 & 1 & -2\end{bmatrix}$ (ii) $A=\begin{bmatrix}2 & 0 & -1\\ 0 & 2 & 0\\ -1 & 0 & 2\end{bmatrix}$ (iii) $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$

7. Find the characteristic equation of the matrix A given below and hence, find the matrix represented by

(i) $A^7-4A^6-20A^5-34A^4-4A^3-20A^2-33A+I$, where $A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$

(ii) $A^6-6A^5+9A^4+4A^3-12A^2+2A-I$, where $A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$

8. $A=\begin{bmatrix}1 & 0\\ 2 & 4\end{bmatrix}$, find eigen values of $4A^{-1}+3A+2I$. (Ans: 9,15)

9. $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & 2\end{bmatrix}$, find eigen values of A^2. (Ans: 1,9,4)

10. $A=\begin{bmatrix}3 & 0\\ 1 & 2\end{bmatrix}$, find eigen values of $A'^2-3A'+4I$. (Ans: 4,2)

11. Two of the eigen values of a 3 x 3 matrix whose determinant is 6 are 1,3. Find the third eigen value. (Ans: 2)

12. The sum of the eigen values of a 3 x 3 matrix is 6 and the product of the eigen values is also 6. If one of the eigen value is one, find other two eigen values. (Ans: 2,3)

13. Find the sum & the product of the eigen values of the matrix

a) $A=\begin{bmatrix}8 & -4\\ 2 & 2\end{bmatrix}$, b) $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$, c) $A=\begin{bmatrix}-2 & -9 & 5\\ -5 & -10 & 7\\ -9 & -21 & 14\end{bmatrix}$

14.Using Cayley-Hamilton Theorem, find matrix represented by

A^7-9A^2+I where $A=\begin{bmatrix}1 & 4\\ 1 & 1\end{bmatrix}$

15Using Cayley-Hamilton Theorem, find matrix represented by

$2A^4-5A^3-7A+6I$ where $A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$

16. Using Cayley-Hamilton Theorem, find matrix represented by

$A^9-6A^8+10A^7-3A^6+A+I$ where $A=\begin{bmatrix}1 & 2 & 3\\ -1 & 3 & 1\\ 1 & 0 & 2\end{bmatrix}$

17. Using Cayley-Hamilton Theorem, find matrix represented by

$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ where $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$

18. Using Cayley-Hamilton Theorem, express matrix B as a quadratic polynomial in A, also find B, where

$B=A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ given $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$

19. Find A^{-1}, A^{-2}, A^{-3} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

20. Find A^{-1}, A^{-2}, A^{3} & A^{4} if $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

TUTORIAL 4

1. Show that the matrix A is diagonalizable. Find the transforming matrix and the diagonal matrix.

(i) $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ ($\lambda = 0,3,15$) (ii) $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ ($\lambda = 1,3,2$)

(iii) $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ (λ = -1,-1,3) (iv) $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ ($\lambda = 0,1,1$)

(v) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (λ = 2,2,8) (vi) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (λ = 5,-3,-3)

(vii) $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (λ = -1,3,4) (viii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (λ = 1,2,2)

(ix) $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (λ = 2,2,1) (x) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (λ = 2,3,6)

2. Show that the matrix $A=\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix.

3. Show that the matrix $A=\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ is not similar to a diagonal matrix.

4. Show that the matrix $A=\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is similar to a diagonal matrix. Also find the transforming matrix and the diagonal matrix.

5. Reduce the following matrix to diagonal form:

(i) $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

6. Show that the following matrices are similar to diagonal matrices. Find the diagonal form and the diagonal matrix:

(i) $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

TUTORIAL 6

1. If $A=\begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3\tan A = A\tan 3$.

2. If $A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find e^{At}.

3. If $A=\begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$, find A^n.

4. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{50}.

5. If $A=\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, find $e^A, 5^A$.

6. If $A=\begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$, find A^{100}.

7. If $A=\begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$, find $\sin A$.

8. Find A^{50} , where i) $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$, ii) $A=\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}$

1. Find A^{100}, where i) $A=\begin{bmatrix}4 & 3\\ 7 & 8\end{bmatrix}$, ii) $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, iii) $A=\begin{bmatrix}1 & 2 & -2\\ 0 & 2 & 1\\ 0 & 0 & -1\end{bmatrix}$
2. Find e^A & 4^A *if* $A=\frac{1}{2}\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$.
3. If $A=\begin{bmatrix}\pi & \frac{\pi}{4}\\ 0 & \frac{\pi}{2}\end{bmatrix}$, find $\cos A$.
4. Show that $\cos O_{3\times3}=I_{3\times3}$

14. Write down the matrix corresponding to each of the following quadratic forms
    (i) $x^2-2y^2+3z^2-2xy-6xz+10yz$ (ii) $x^2-2y^2+3z^2-4xy+xz-2yz$
    (iii) $2x_1^{\ 2}-3x_2^{\ 2}+4x_3^{\ 2}+x_4^{\ 2}-2x_1x_2+3x_3x_1-4x_1x_4-5x_2x_3+6x_2x_4+x_3x_4$

15. Reduce the quadratic form $2x_1x_2+2x_2x_3+2x_3x_1$ into canonical form. Examine for definiteness.
16. Find the matrix of the quadratic form $6x_1^{\ 2}+3x_2^{\ 2}+3x_3^{\ 2}-4x_1x_2+4x_3x_1-2x_2x_3$ and find the linear transformation $\vec{X}=Q\vec{Y}$ which transforms the given form to sum of squares. Write also rank, index, signature and nature of the quadratic form.

17. Reduce $8x^2+7y^2+3z^2-12xy+4xz-8yz$ into canonical form by orthogonal transformation.
18. Reduce the following quadratic form $6x_1^{\ 2}+3x_2^{\ 2}+14x_3^{\ 2}+4x_1x_2+18x_3x_1+4x_2x_3$ to diagonal form through congruent transformations.
19. Reduce the following quadratic form to canonical form and find its rank and signatures. Also write linear transformation which brings about the normal reduction:
    $$21x_1^{\ 2}+11x_2^{\ 2}+2x_3^{\ 2}-30x_1x_2+12x_3x_1-8x_2x_3.$$
20. Reduce the following quadratic form to sum of squares and interpret your result:
    $$3x_1^{\ 2}+2x_2^{\ 2}+x_3^{\ 2}+4x_1x_2-2x_3x_1+6x_2x_3$$

21. Reduce the following quadratic form to **canonical form** using ***congruent transformation***. Also write ***linear transformation***. State ***rank, index , signature & nature of quadratic form***

a) $3x_1^{\ 2}+4x_1x_2+2x_2^{\ 2}+x_3^{\ 2}-2x_1x_3+6x_2x_3$, find non-zero values of x_1, x_2, x_3 which will make the quadratic form positive and negative respectively.

b) $x_1^2+2x_1x_2+2x_2^2+3x_3^2-2x_3x_1+2x_2x_3$, find non-zero values of x_1, x_2, x_3 which will make the quadratic form positive and negative respectively.

c) $5x_1^2+6x_1x_2+26x_2^2+10x_3^2+14x_3x_1+4x_2x_3$, find non-zero values of x_1, x_2, x_3 which will make the quadratic form zero.

d) $x_1^2-2x_1x_2+2x_2^2+2x_3^2+x_3x_1-2x_2x_3$

e) $21x_1^2-30x_1x_2+11x_2^2+2x_3^2+12x_3x_1-8x_2x_3$, find non-zero values of x_1, x_2, x_3 which will make the quadratic form zero.

f) $2x_1^2+12x_1x_2+x_2^2-3x_3^2-4x_3x_1-8x_2x_3$

g) $6x_1^2-4x_1x_2+3x_2^2+3x_3^2+4x_3x_1-2x_2x_3$

h) $10x_1^2-4x_1x_2+2x_2^2+5x_3^2-10x_3x_1+6x_2x_3$

i) $4x_1^2-8x_1x_2+3x_2^2+x_3^2+4x_3x_1-6x_2x_3$

j) $x_1^2-2x_1x_2-3x_2^2+4x_3x_1-12x_2x_3$, find non-zero values of x_1, x_2, x_3 which will make the quadratic form positive.

k) $6x_1^2-4x_1x_2+3x_2^2+3x_3^2+4x_3x_1-2x_2x_3$

l) $6x_1^2+4x_1x_2+3x_2^2+14x_3^2+18x_3x_1+4x_2x_3$

m) $2x_1x_2+2x_3x_1+2x_2x_3$

n) $x_1x_2+x_3x_1+x_2x_3$

o) $2x_1x_2+4x_3x_1+6x_2x_3$

22. Reduce the following quadratic form to **canonical form using orthogonal transformation**. Also write ***linear transformation***. State ***rank, index , signature & nature of quadratic form***.

a) $3x_1^2-2x_1x_2+5x_2^2+3x_3^2+2x_3x_1-2x_2x_3$

b) $7x_1^2+5x_2^2+6x_3^2-4x_3x_1-4x_2x_3$

c) $6x_1^2-4x_1x_2+3x_2^2+3x_3^2+4x_3x_1-2x_2x_3$

d) $x_1^2+4x_1x_2+4x_2^2+9x_3^2+6x_3x_1+12x_2x_3$

e) $7x_1^2+8x_1x_2-8x_2^2-8x_3^2-8x_3x_1-2x_2x_3$

f) $3x_1^2+4x_1x_2+3x_3^2+8x_3x_1+4x_2x_3$

g) $10x_1^2-4x_1x_2+2x_2^2+5x_3^2-10x_3x_1+6x_2x_3$

h) $2x_1x_2+2x_3x_1-2x_2x_3$

i) $x_1^2+2x_2^2+4x_1x_2+3x_3^2+4x_2x_3$

j) $x_1^2+3x_2^2+3x_3^2-2x_2x_3$

k) $8x_1^2+7x_2^2+12x_1x_2+3x_3^2-8x_2x_3+4x_1x_3$

l) $2\left(x_1^{\,2} + x_2^{\,2} + x_2 x_1\right)$

m) $2\left(x_1 x_2 + x_3 x_1 + x_2 x_3\right)$

n) $17x_1^{\,2} + 17x_2^{\,2} - 30x_2 x_1$

TUTORIAL 7

Find the Laplace transforms of the following functions:

1. $f(t) = \begin{cases} \frac{t}{T}, & 0 < t < T \\ 1, & t > T \end{cases}$

2. $f(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$

3. $f(t) = \begin{cases} t^2, & 0 < t < 1 \\ 1, & t > 1 \end{cases}$

4. $f(t) = \begin{cases} t, & 0 < t < a \\ b, & t > a \end{cases}$

Find the Laplace transforms of the following functions:

5. $\cos^3 2t$ 6. $\sinh^2 2t$ 7. $(\sin t - \cos t)^2$ 8. $5e^{-t/2} + t^{-1/2} + 7\sin\left(t/2\right)$

9. $\sin(\varpi t + \alpha)$ 10. $\sin^5 t$ 11. $\sin\sqrt{t}$ 12. $\cos^5 t$ 13. $\sinh^5 t$

14. $\cosh^5 t$ 15. $\frac{\cos\sqrt{t}}{\sqrt{t}}$ 16. If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)dt = \frac{1}{4}$, then find α.

17. $e^{4t}\sin^3 t$ 18. $\cosh 2t \cdot \cos 2t$ 19. $\sinh at \cdot \sin at$ 20. $\sinh\left(t/2\right)\sin t/2$

21. $\frac{\cosh 2t \cdot \sin t}{e^t}$ 22. $e^{-t}\sinh t \cdot \sin t$ 23. $\sin 2t \cdot \cos t \cdot \cosh 2t$

24. $\cos t \cdot \cos 2t \cdot \cos 3t$ 25. $e^{2t}\sin^4 t$ 26. $t^2 \sin at$ 27. $te^{-4t}\sin 3t$

28. $t\cos(3t+4)$ 29. $\frac{\sin^2 t}{t}$ 30. $\frac{\sinh t}{t}$ 31. $\frac{\cos 2t - \cos 3t}{t}$

32. Evaluate: $\int_0^\infty e^{-2t}\sin^3 t\,dt$ 33. Evaluate: $\int_0^\infty e^{-t}t^3 \sin t\,dt$ 34. Evaluate: $\int_0^\infty e^{-3t} t \sin t\,dt$

35. Evaluate: $\int_0^\infty e^t \frac{\sin t}{t}\,dt$

TUTORIAL 8

Find the inverse Laplace transforms of the following functions:

1. $\frac{2}{s}+\frac{1}{s^3}+\frac{1}{s+4}$ 2. $\frac{2s+3}{s^2+9}$ 3. $\frac{4s+15}{16s^2-25}$ 4. $\frac{2s+3}{s^2+2s+2}$ 5. $\frac{s+2}{s^2+4s+7}$

6. $\frac{s+2}{(s+3)(s+1)^3}$ 7. $\frac{2s}{s^4+4}$ 8. $\frac{s}{s^4+s^2+1}$ 9. $\frac{1}{s^4-2s^3}$ 10. $\frac{2s-\pi}{s^3(s-\pi)}$

11. Find the inverse Laplace transforms by using convolution theorem:

(i) $\frac{1}{s\,(s+a)}$ (ii) $\frac{1}{s\,(s+a)^2}$ (iii) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

(iv) $\frac{s}{(s^2+a^2)(s^2+b^2)}$ (v) $\frac{1}{(s+a)(s+b)^2}$ (vi) $\frac{(s+2)^2}{(s^2+4s+8)^2}$

(vii) $\frac{1}{(s+3)(s^2+2s+2)}$ (viii) $\frac{s^2}{\left(s^2+a^2\right)^2}$ (ix) $\frac{s^2}{\left(s^2-a^2\right)^2}$

(x) $\frac{1}{(s-3)(s+3)^2}$ (xi) $\frac{s^2}{\left(s^2+4\right)\left(s^2+1\right)}$ (xii) $\frac{s^2+2s+3}{\left(s^2+2s+2\right)\left(s^2+2s+5\right)}$

(xiii) $\frac{1}{s^2(s+1)^2}$ (xiv) $\frac{1}{\left(s^2+4s+13\right)^2}$ (xv) $\frac{(s+3)^2}{\left(s^2+6s+5\right)^2}$

TUTORIAL 9

1. Find Laplace transform of $f(t)=K\frac{t}{T}$ *for* $0<t<T$ and $f(t)=f(t+T)$.

2. Find the Laplace transform of $f(t)=\begin{cases}1, & 0\le t<a\\ -1, & a<t<2a\end{cases}$ and $f(t)$ is periodic function with period $2a$.

3. Find Laplace transform of $f(t)=\begin{cases}a\sin pt, & 0<t<\pi/p\\ 0, & \pi/p<t<2\pi/p\end{cases}$ and $f(t)=f\left(t+2\pi/p\right)$.

4. Find Laplace transform of $f(t)=\begin{cases}E, & 0\le t\le p/2\\ -E, & p/2\le t\le p\end{cases}$, $f(t+p)=f(t)$.

5. Find Laplace transform of $\sin t\; H\left[t-\left(\pi/2\right)\right]-H\left[t-\left(3\pi/2\right)\right]$.

6. Find Laplace transform of $\left(1+2t-3t^2+4t^3\right)H(t-2)$.

7. Find Laplace transform of $\left(1+3t-4t^2+2t^3\right)H(t-3)$.

8. Find inverse Laplace transform of the following:

(i) $\dfrac{e^{-as}}{(s+b)^{5/2}}$ (ii) $\dfrac{e^{-4-3s}}{(s+4)^{5/2}}$ (iii) $\dfrac{e^{-\pi s}}{s^2-2s+2}$ (iv) $\dfrac{e^{-3s}}{(s+4)^3}$

(v) $\dfrac{e^{-s}\left(1+\sqrt{s}\right)}{s^3}$ (vi) $\dfrac{s\,e^{-s/2}+\pi\,e^{-s}}{s^2+\pi^2}$ (vii) $\dfrac{s\,e^{-as}}{s^2+b^2}$

9. 20. Find Laplace transform of :

(i) $\sin 2t\,\delta(t-2)$ (ii) $t[H(t-4)]+t^2\delta(t-4)$

(iii) $\sin 2t\,\delta\left(t-\dfrac{\pi}{2}\right)-t^2\delta(t-2)$ (iv) $t^4[H(t-2)]+t^2\delta(t-2)$

10. Find: (i) $L^{-1}\left(\dfrac{s}{s+1}\right)$ (ii) $L^{-1}\left(e^{-as}\sin a\right)$

11. Evaluate: (i) $\int_0^\infty e^{-t}\left(1+2t-3t^2+4t^3\right)H(t-2)\,dt$

(ii) $\int_0^\infty e^{-2t}\left(1+t+t^2\right)H(t-3)\,dt$

12. Find Laplace Transform of $tH(t-2)$
13. Find Laplace Transform of $t^2H(t-3)$
14. Find Laplace Transform of $t^4H(t-3)$
15. Find Laplace Transform of $\sin tH(t-\pi)$
16. Find Laplace Transform of $g(t)=(t-2)^2, t>2$ and $g(t)=0, 0<t<2$
17. Find Laplace Transform of $\sin t.H\left(t-\dfrac{\pi}{2}\right)-H\left(t-\dfrac{3\pi}{2}\right)$
18. Find Laplace Transform of $(1+2t-3t^2+4t^3).H(t-2)$
19. Find Laplace Transform of $(1+3t-4t^2+2t^3).H(t-3)$
20. Find Laplace Transform of $f(t)=(t-3)^4, t>3$ and $f(t)=0, 0<t<3$
21. using Laplace Transform $\int_0^\infty e^{-t}(1+2t-3t^2+4t^3).H(t-2)dt$
22. using Laplace Transform $\int_0^\infty e^{-t}(1+2t-t^2+t^3).H(t-1)dt$
23. Find Laplace Transform of $f(t)=\begin{cases}1+2t-t^2, t>2\\ 0, 0<t<2\end{cases}$
24. Find:

(i) $L^{-1}\left[\frac{e^{4-3s}}{(s+4)^{5/2}}\right]$ (ii) $L^{-1}\left[\frac{(s+1)e^{-s}}{s^2+s+1}\right]$ (iii) $L^{-1}\left[\frac{8e^{-3s}}{s^2+4}\right]$

(iv) $L^{-1}\left[\frac{e^{-5s}}{(s-2)^4}\right]$ (v) $L^{-1}\left[\frac{e^{-\pi s}}{s^2+9}\right]$ (vi) $L^{-1}\left[\frac{se^{-as}}{s^2+3s+2}\right]$

(vii) $L^{-1}\left[e^{-s}\left(\frac{1-\sqrt{s}}{s^2}\right)^2\right]$ (viii) $L^{-1}\left[e^{-s}\left(\frac{1+\sqrt{s}}{s^3}\right)\right]$ (ix) $L^{-1}\left[\frac{se^{-as}}{s^2+b^2}\right]$

(x) $L^{-1}\left[\frac{se^{-3s}}{s^2-1}\right]$ (xi) $L^{-1}\left[\frac{e^{-bs}}{s^2(s+a)}\right]$ (xii) $L^{-1}\left[\frac{e^{-\pi s}}{s^2(s^2+1)}\right]$

(xiii) $L^{-1}\left[\frac{e^{-4s}}{\sqrt{2s+7}}\right]$ (xiv) $L^{-1}\left[\frac{se^{-2s}}{s^2+2s+2}\right]$ (xv) $L\left[\sin 3t.\delta\left(t-\frac{\pi}{3}\right)\right]$

(xvi) $L[\sin 3t.\delta(t-4)]$ (xvii) $L\left[\cos t.\delta\left(t-\frac{\pi}{2}\right)+\left(t^3-4t+3\right)\delta(t-5)\right]$

(xviii) $L[t^2.\delta(t-4)+t.H(t-4)]$ (xix) $L[t^4.\delta(t-2)+t^2.H(t-2)]$

(xx) $L\left[\sin t.\delta\left(t-\frac{\pi}{2}\right)-t^2.\delta(t-2)\right]$ (xxi) $L^{-1}\left[\frac{e^{-s}}{(s+1)(s-2)^2}\right]$ (xxii) $L^{-1}\left[\frac{se^{-3s}}{(s^2-a^2)^2}\right]$

TUTORIAL 10

Application Of Laplace Transforms To Solve Differential Equations :-

Solve:

1. $3\frac{dy}{dt}+2y=e^{3t}, y=1 \quad at \quad t=0$.

2. Solve: $L\frac{dI}{dt}+RI=Ee^{-at}$, $y=1$ *where* $I(0)=0$.

3. Solve: $\frac{dy}{dt}+3y=2+e^{-t}$, *if* $y=1$ *at* $t=0$.

4. Solve: $(D^2-3D+2)y=4e^{2t}$ with $y(0)=-3$ *and* $y'(0)=5$.

5. Solve: $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$, when $t=0$, $y=0$ *and* $\frac{dy}{dt}=0$.

6. Solve $(D^2-3D+2)y=4e^{2t}$ with $y(0)=-3$ and $y'(0)=-3$

7. Solve $(D^2-D-2)y=20\sin 2t$ with $y(0)=1$ and $y'(0)=2$

8. Solve $(D^2+3D+2)y=2(t^2+t+1)$ with $y(0)=2$ and $y'(0)=0$

9. Solve $(D^2+2D+5)y=e^{-t}\sin t$ with $y(0)=0$ and $y'(0)=1$

10. Solve $(D^3-2D^2+5D)y=0$ with $y(0)=0$, $y'(0)=0$ and $y''(0)=1$

11. Solve $\frac{d^2y}{dt^2}+9y=\delta(t)$ given that $y=0$, $\frac{dy}{dt}=0$ at $t=0$

12. Solve $\frac{d^2y}{dt^2}+16y=\delta(t)$ given that $y=0$, $\frac{dy}{dt}=0$ at $t=0$

13. Solve $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=t\delta(t-1)$ given that $y=0$, $\frac{dy}{dt}=0$ at $t=0$

14. Solve $\frac{d^2y}{dt^2}+4y=f(t)$ given that $y=0$, $\frac{dy}{dt}=1$ at $t=0$

Where (i) $f(t)=\begin{cases}1 & \text{when } 0<t<1\\ 0 & \text{when } t>1\end{cases}$

(ii) $f(t)=H(t-2)$

15. Solve $\frac{dx}{dt}+y=\sin t, \frac{dy}{dt}+x=\cos t$;where x = 0 , y = 2 at t = 0

16. Solve $Dx-2y-x=-2te^{-t}+e^{t}-6t, D^2x-Dy=-te^{-t}-2e^{-t}-3$; where x = 0 ,

17. y = 0 & Dx = 1 at t = 0

<u>TUTORIAL 11</u>

Find the Fourier series for the following functions:

1. $f(x)=e^{-x}$ in $(0, 2\pi)$.

2. $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in $(0, 2\pi)$. Hence deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+$..........

3. $f(x)=x^3$ in $(0, 2\pi)$ also in $(-\pi, \pi)$.

4. $f(x) = e^x$ in $(-\pi, \pi)$.

5. $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$.

6. $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$ prove that $f(x) = \dfrac{1}{\pi} + \dfrac{1}{2}\sin x - \dfrac{2}{\pi}\sum_{n=1}^{\infty} \dfrac{\cos 2nx}{4n^2 - 1}$.

Hence show that $\dfrac{1}{1.3} - \dfrac{1}{3.5} + \dfrac{1}{5.7} - .... = \dfrac{\pi - 2}{4}$.

7. $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

8. $f(x) = \begin{cases} \dfrac{x}{2}, & 0 < x < \pi \\ \dfrac{x}{2} - \pi, & \pi < x < 2\pi \end{cases}$

9. $f(x) = \begin{cases} a, & 0 < x < \pi \\ -a, & \pi < x < 2\pi \end{cases}$

10. $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi, & 0 < x < \pi \end{cases}$

11. $f(x) = x^2$ in $(0, a)$.

12. $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$

13. $f(x) = x + x^2$ in $-1 < x < 1$.

14. $f(x) = \pi^2 - x^2$ in $-\pi \le x \le \pi$.

15. $f(x) = 1 - x^2$ in $-1 \le x \le 1$.

16. $f(x) = \begin{cases} x + \dfrac{\pi}{2}, & -\pi < x < 0 \\ \dfrac{\pi}{2} - x, & 0 < x < \pi \end{cases}$, deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ...$.

Using parseval's identity prove that $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + ...$

17. $f(x) = x\cos x$ in $(-\pi, \pi)$.

18. $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$

19. $f(x) = x - x^2$ in $-1 < x < 1$.

20. $f(x) = \pi + x, \quad -\pi < x < 0$
$\quad = \pi - x, \quad 0 < x < \pi$

21. $f(x) = \dfrac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$. Hence deduce that $\dfrac{\pi^2}{6} = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + ...$

22. $f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1 + x, & -1 < x < 0 \\ 1 - x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$

TUTORIAL 12

1. Find half range cosine series for $f(x) = x$ in $0 < x < 2$. Using parseval's identity, deduce that (i) $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + ...$ (ii) $\dfrac{\pi^4}{90} = \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + ...$

2. Find half range sine series for $f(x) = x\sin x$ in $(0, \pi)$.

3. Expand $f(x) = \begin{cases} kx, & 0 < x < l/2 \\ 0, & l/2 < x < l \end{cases}$ into half range cosine series.

   Deduce the sum of the series $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ...$

4. Find half range cosine series for $f(x) = \begin{cases} 1, & 0 < x < a/2 \\ -1, & a/2 < x < a \end{cases}$.

5. Obtain a half range cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l - x), & l/2 \le x \le l \end{cases}$.

   Deduce the sum of the series $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ...$

6. Find a half range cosine series to represent $f(x) = \sin x$ in $0 \le x \le \pi$.

7. Obtain a half range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$.

8. Obtain a half range sine series for $f(x) = \begin{cases} \left(\frac{1}{4}\right) - x, & 0 < x < \left(\frac{1}{2}\right) \\ x - \left(\frac{3}{4}\right), & \left(\frac{1}{2}\right) < x < 1 \end{cases}$.

9. Find half range cosine series for $f(x) = a\left\{1 - \frac{x}{l}\right\}$ $0 < x < l$.

10. Obtain a half range sine series for $f(x) = x(2 - x)$ in $0 < x < 2$.

11. Show that a constant c can be expanded in a infinite series $\frac{4c}{\pi}\left\{\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + ...\right\}$ in the range $0 < x < \pi$.

12. Find half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. Hence show that $\pi^2 = 8\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\right)$.

13. Obtain a half range sine series for $f(t) = t - t^2,\ 0 < t < 1$.

14. Find the half range sine series for $f(x) = x\cos x$ in $(0, \pi)$.

15. Obtain the half range sine series for e^x in $0 < x < 1$.

16. Expand $f(x) = \begin{cases} kx, & 0 < x < l/2 \\ 0, & l/2 < x < l \end{cases}$ into half range cosine series. Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$

17. Show that the set of functions

$\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), ...$ is orthogonal over $(0, L)$.

18. Show that the set of functions

$1,\ \sin\frac{\pi x}{L},\ \cos\frac{\pi x}{L},\ \sin\frac{2\pi x}{L},\ \cos\frac{2\pi x}{L}, ...$

form an orthogonal set in $(-L, L)$ and construct an orthonormal set.

19. Prove that $f_1(x)=1,\ f_2(x)=x,\ f_3(x)=\left(3x^2-1\right)/2$ are orthogonal over $(-1,1)$.

20. Show that the set of functions $1, \sin\frac{\pi x}{L}, \cos\frac{\pi x}{L}, \sin\frac{2\pi x}{L}, \cos\frac{2\pi x}{L}, ...$ form an orthogonal set in $(-L, L)$ and construct an orthonormal set.

21. Show that the set of functions $\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), ...$ is orthogonal over $(0, L)$.

22. Show that the set of functions $\frac{\cos x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\cos 3x}{\sqrt{\pi}}, ...$ form an orthonormal set in the interval $(-\pi, \pi)$.

23. Show that the set of functions $\frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \frac{\sin 3x}{\sqrt{\pi}}, ...$ form an orthonormal set in the interval $(-\pi, \pi)$.

24. Prove that the set of functions $\sin x, \sin 2x, \sin 3x, ...$ is orthogonal over $[0, 2\pi]$. Hence construct orthonormal set of functions.

25. Show that the set of functions $\cos x, \cos 2x, \cos 3x, ...$ is orthogonal over $[-\pi, \pi]$. Hence construct orthonormal set of functions.

26. Show that the set of functions $\sin x, \sin 2x, \sin 3x, ...$ is orthogonal over $[0, \pi]$.

27. Show that the set of functions $\cos x, \cos 3x, \cos 5x, ...$ is orthogonal over $\left(0, \pi/2\right)$.

28. Solve: $y'' - y = t,\ y(0) = y'(0) = 1$.

14. Solve: $\frac{dx}{dt} + 5x - 2y = t, \quad \frac{dy}{dt} + 2x + y = 0, \quad x = y = 0 \quad at \quad t = 0.$

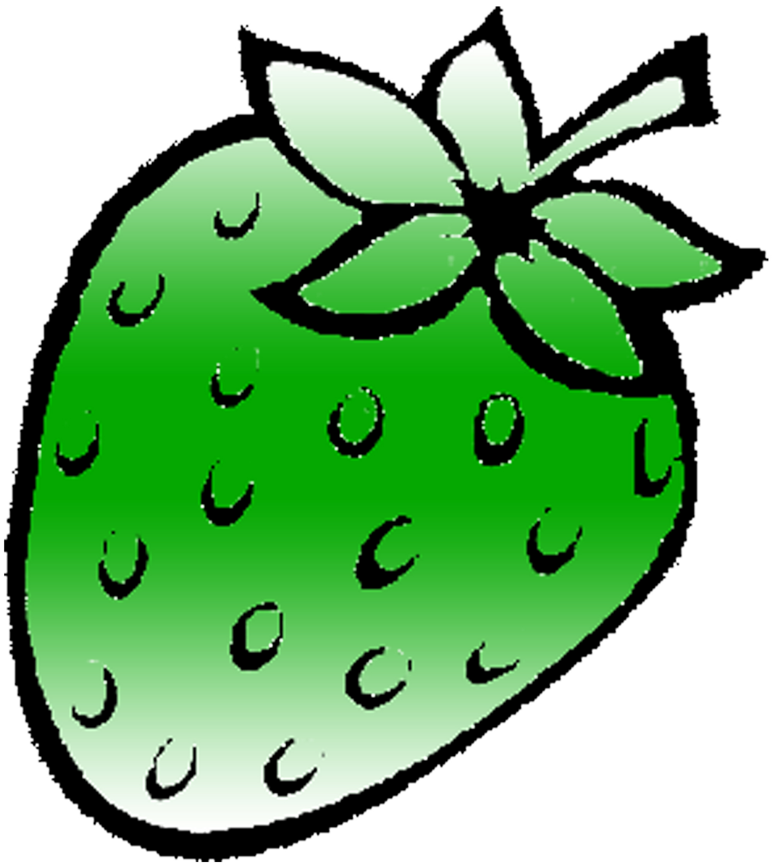
15. Solve: $y''' + 2y'' - y' - 2y = 0,\ y(0) = y'(0) = 0\ and\ y''(0) = 6.$

16. Solve: $y'' + 5y' + 4y = 3 \cdot \delta(t-2) \quad at \quad t = 0, y(0) = 2\ \&\ y'(0) = -2.$

17. Solve: $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = t\delta(t-1), \quad y(0) = 0,\ y'(0) = 0.$

18. Solve: $\frac{d^2y}{dt^2} + 4y = f(t), \quad y(0) = 0,\ y'(0) = 1 \quad \& \quad f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$

STRAWBERRY



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